Legacy vs. SIGMET dBZ Calculation

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In this paper, we discuss how the weather radar equation is interpreted between legacy and SIGMET. This difference shows up in the different ranges for SYSCAL (legacy) and dBZ_0 (SIGMET), but model the exact same equation. This paper will show how each is derived, and give nominal values for each.

The legacy values shown are well validated; however, we still have to validate the SIGMET numbers shown on the WSR-88D through calculations of dBZ_0.

Radar Equation:

The basic weather radar equation makes the assumption of Gaussian returns with Rayleigh scattering (a large number of small points in the radiated volume).

The general weather radar equation for Z_{en} is (using standard metric units and organized in a "NEXRAD friendly" format):

$$Z_e = P_R \times R^2 \times \frac{1}{L_P} \times \frac{2^{10} \times \ln(2) \times \lambda^2}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L}$$

Figure 1, Radar Equation

Z is normally cast as $\frac{mm^6}{m^3}$ so Z_{en} needs to be converted from m^3 to those units (a factor of 10^{18}).

The main difference between legacy and SIGMET is how they handle the P_R, the receive power. In the radar equation, P_R is Signal only. However, in the radar return the signal processor sees, noise is mixed with the Signal and must be accounted for to get an accurate Z_{en}. Also, since we are using A/D converters on the signal voltage (not power) and our receiver changes the gain of P_R, these must also be accounted for (a^2g accounts for this in legacy). SIGMET's IQ values correct for a^2g.

The legacy system uses P_R as (Return - Noise), while Signet's model for P_R is (Return – Noise)/Noise, the signal to noise ratio. This difference is accounted for in the calibration constants used by legacy (SYSCAL) and SIGMET (dBZ_0).

The effect of this is that Signet's calibration constant, dBZ_0, differs from SYSCAL by the Receiver Noise value (around -57dB in the legacy WSR-88D), the difference in receiver gain, and the difference in the a^2g term.

Table 1 shows the parameters associated with this paper, and the common units these parameters are displayed in. The radar equation used assumes standard metric units, so to use these parameters they must be converted to our common units first (i.e. cm converted to m for the radar equation).

Table 1, Parameters

Symbol	Name	Units
λ	Wavelength	cm
π	Pi	Unitless

K	Refractivity	Unitless
P_R	Receive Power	mW
P_T	Transmit Power	kW
N	Noise	mW
R	Range	km
L	Losses (except propagation loss, see table 2)	Unitless
τ	Pulsewidth	μsec
G	Antenna Gain	Unitless
c	Speed of Light	m/sec
K	Refractivity	Unitless
ln	Natural Logarithm	Unitless
L_P	2 way atmospheric propagation loss	Unitless
g	Receiver Gain	Unitless
θ	Beamwidth	Radians

Table 2, Losses

Symbol	Name	Description	Nominal Value (dB)
L_t	Transmitter	Transmitter Waveguide Loss	2.5
L_r	Receiver	Receiver Waveguide Loss	.63
L_d	Detection	Receiver Detection Loss	1.5
g	Receiver Gain	Receiver gain from Receiver Protector to A/D conversion	Legacy: 55dB ORDA: 39dB

Legacy Casting:

$$dBZ = 10\log(P_R - N) + 20\log(R) - 10\log(L_P) + SYSCAL$$

$$SYSCAL = 10\log\left(\frac{2^{10} \times \ln(2) \times \lambda^2 \times 10^{18} \times 10^6 \times 10^{-3}}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L_t \times L_r \times L_d} \times \frac{1}{a^2 g}\right)$$

SIGMET Casting:

$$dBZ = 10\log\left(\frac{P_R - N}{N}\right) + 20\log(R) - 10\log(L_P) + dBZ_0$$

To calculate dBZ_0, we calculate the radar equation for signal power equal to the noise power at 1km. We'll put this into Figure 1, Radar Equation with the following definitions:

$$P_R = \left(\frac{2N - N}{N}\right) = 1 \quad \text{(signal power equal to noise power)}$$

Since P_R has Noise in the denominator, it must be in the numerator somewhere in the radar equation as well
R=1km (since we cast R in terms of km, this will cancel out the range)
L_P=1 (since we're using a test signal to calculate dBZ_0, there's no correction for atmospheric gas)

Unit changes: Z conversion to $\frac{mm^6}{m^3}$ is 10^{18}, P_R conversion to mW is 10^{-3} and m^2 conversion to km^2 is 10^6.

Z_0 becomes:

$$Z_0 = \left(\frac{2N - N}{N}\right) \times 1^2 \times \frac{1}{1} \times \frac{2^{10} \times \ln(2) \times \lambda^2}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L} \times \left(10^{18} \times 10^{-3} \times 10^6\right) \times N$$

Since L is equal to the product of the losses, and our reference point for measurements is the input to the Receiver Protector, L is split up as follows:

$$L = (L_t \times L_r \times L_d) \times g$$

Now we convert Z to dBZ:

$$dBZ_0 = 10\log(P_R) + 20\log(R_0) - 10\log(L_P) + 10\log\left(\frac{2^{10} \times \ln(2) \times \lambda^2 \times 10^{18} \times 10^{-3} \times 10^6}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L_t \times L_r \times L_d} \times \frac{N}{g}\right)$$

$$dBZ_0 = 10\log\left(\frac{2^{10} \times \ln(2) \times \lambda^2 \times 10^{18} \times 10^{-3} \times 10^6}{\pi^3 \times P_T \times G^2 \times \theta^2 \times c \times \tau \times |K|^2 \times L_t \times L_r \times L_d} \times \frac{N}{g}\right)$$

SYSCAL – dBZ_0 Comparison

The difference between SIGMET and legacy in the radar equation is how P_R is handled. In the constants, the $\frac{N}{a^2 g}$ term is the difference between legacy and SIGMET calibration constants. The g term (receiver gain from front end to A/D conversion) is used here because P_R is referenced to this point (front end insertion).

SYSCAL interpretation:

These terms are related to how P_R is represented in the radar equation. For legacy, the P_R term is in digits^2 from the A/D converter without being converted to voltage.

The g term is the same for both, representing the receiver gain. In legacy, the a^2 term represents conversion from A/D digits to power (voltage^2 represents power, and since a represents volts/digit, a^2 represents power/digit^2).

dBZ_0 interpretation:

dBZ_0 is used as a signal reference that all reflectivity is compared against. It represents the dBZ of a signal 3dB above the noise (Signal=Noise) at a range of 1km.

For SIGMET, I and Q values are converted to voltage before pulse pair and power calculations. Therefore dBZ_0 does not have to account for the a^2 term.

SIGMET models P_R as the Signal to Noise ratio, therefore putting N in the calibration constant.

Calculations of SYSCAL and dBZ_0

Table 3 shows the calculations of SYSCAL and dBZ_0 using typical values for various components. Numerator values are above the heavy line (at Noise value), and denominator values are below. In dB, this changes the sign. The column "Radar Equation Value" shows the number needed for the radar equation. This takes into account where we need exponentiation of a value (for example, the radar equation uses the wavelength (λ) squared, and the Radar Equation Value column is the square of the value column). Refer to the Radar Equation above to determine the exponentiation used.

Table 3, SYSCAL and dBZ0

Term	Units	Value	Radar Equation Value	dB	SYSCAL	dBZ0
2^10	Unitless	1024.00	1024.00	30.10	30.10	30.10
ln(2)	Unitless	0.693147	0.693147	-1.59	-1.59	-1.59
wavelength	cm	10	100	20.00	20.00	20.00
cm to m conversion	m	0.01	0.0001	-40.00	-40.00	-40.00
m3 to mm6/m3 conversion		1.00E+18	1.00E+18	180.00	180.00	180.00
milliwatts to W conversion	W	0.001	0.001	-30.00	-30.00	-30.00
km to m conversion	m	1000	1000000	60.00	60.00	60.00
Noise	mW			-78		-78
antenna gain	Unitless			-91.60	-91.60	-91.60
antenna beamwidth	degrees	0.91	0.8281	0.82	0.82	0.82
degrees to radians conversion	radians	0.017453	0.000305	35.16	35.16	35.16
Pulsewidth	Microseconds	1.57	1.57	-1.96	-1.96	-1.96
microseconds to seconds conversion	sec	0.000001	0.000001	60.00	60.00	60.00
speed of light	m/sec	3.00E+08	3.00E+08	-84.77	-84.77	-84.77
Refractivity^2	Unitless	0.93	0.93	-.315	.32	.32
pi	Unitless	3.14159	31.0062	-14.91	-14.91	-14.91
transmitted power	kW	700	700	-28.45	-28.45	-28.45
kW to W conversion	W	1000	1000	-30.00	-30.00	-30.00
transmitter waveguide loss	Unitless			2.25	2.25	2.25
receiver waveguide loss	Unitless			0.63	0.63	0.63
receiver detection loss	Unitless			1.5	1.5	1.5
a2g (legacy)	Unitless			-57.00	-57.00	
receiver gain (SIGMET)	Unitless			-38.75		-35.75
Total					10.50	-46.26

The SYSCAL value 10.50 agrees well with the field measured SYSCAL of 10.627 (average SYSCAL from all field systems before installation of EMI Filter, standard deviation is 1.10).

The SIGMET receiver gain is the gain from the Receiver Protector 2A3J1 to the input of the SIGMET IFD, including the 2db insertion loss from Signet's anti-aliasing filter and a 3dB attenuator between the Mixer/Preamp and the IFD. This differs from legacy significantly because of removal of the IF and analog portions of the legacy receiver (the gain is just to the output of the Mixer/Preamplifier 4A5)

Conclusion:

Both legacy and SIGMET use the same equation for reflectivity, they just model it differently. All the terms are accounted for, and therefore there will be no difference in reflectivity between legacy and SIGMET.